Section 2.1

(14) $\lim_{t \to 7^+} \frac{\sqrt{2(t-7)^3}}{t-7} = \lim_{t\to 7^+} \sqrt{2(t-7)} = 0$

$(t>7)$

(18) $\lim_{h\to 0} \frac{[(x+h)^2 - x^2]}{h} = \lim_{h\to 0} \frac{(x^2+2xh+h^2) - x^2}{h} = \lim_{h\to 0}(2x+h) = 2x$

(34) Consider $g(x) = \begin{cases} -x+1 & \text{for } x<1 \\ x-1 & \text{for } 1<x<2 \\ 5-x^2 & \text{for } x\geq 2 \end{cases}$.

Then (a) $\lim_{x\to 1^+} g(x) = \lim_{x\to 1^+}(x-1) = 0$ and $\lim_{x\to 1^-} g(x) = \lim_{x\to 1^-}(-x+1) = 0$

$(x>1)$ $(x<1)$

$\Rightarrow \lim_{x\to 1} g(x) = 0$

(b) $g(1)$ is not defined

(c) $\lim_{x\to 2^+} g(x) = \lim_{x\to 2^+}(5-x^2) = 5-(2)^2 = 1$

$(x>2)$

$\lim_{x\to 2^-} g(x) = \lim_{x\to 2^-}(x-1) = 1$

$(x<2)$

$\Rightarrow \lim_{x\to 2} g(x) = 1$

(46) (a) $\lim_{x\to 3^+} \frac{[[x]]}{x} = \lim_{x\to 3^+} \frac{3}{x} = 1$ and $\lim_{x\to 3^-} \frac{[[x]]}{x} = \lim_{x\to 3^-} \frac{2}{x} = \frac{2}{3}$

$\Rightarrow \lim_{x\to 3} \frac{[[x]]}{x}$ doesn't exist

(b) $\lim_{x\to 0^+} \frac{[[x]]}{x} = \lim_{x\to 0^+} \frac{0}{x} = 0$

(c) $\lim_{x\to 1.8} [[x]] = 1$

(d) $\lim_{x\to 1.8} \frac{[[x]]}{x} = \lim_{x\to 1.8} \frac{1}{x} = \frac{1}{1.8} \left(= \frac{5}{9}\right)$

(57) $\lim_{x\to 2^-} \frac{x^2-x-2}{|x-2|} = \lim_{x\to 2^-} \frac{\cancel{(x-2)}(x+1)}{-\cancel{(x-2)}} = \lim_{x\to 2^-} -(x+1) = -3$

$(x<2)$

Section 2.3

(24) $\lim_{w\to -2} \frac{\overset{\to 0}{(w+2)}\overset{\to 0}{(w^2-w-6)}}{[w^2+4w+4]_{\to 0}} = \lim_{w\to -2} \frac{(w+2)(w-3)(w+2)}{(w+2)^2} = \lim_{w\to -2} (w-3) = -5$

(27) $\because \lim_{x\to a} f(x) = 3$ and $\lim_{x\to a} g(x) = -1$

$\therefore \lim_{x\to a} \sqrt[3]{g(x)}\{f(x)+3\} = \sqrt[3]{(-1)}\cdot\{3+3\} = -6$

(42) $\lim_{x\to(-\pi)^+} \frac{\sqrt{\pi^3+x^3}}{x} = \frac{\sqrt{\pi^3+(-\pi)^3}}{-\pi} = 0$

(47) $\lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{(-x)} = \lim_{x\to 0^-}(-1) = -1$

(49) Prove that $\lim_{x\to a} f(x)$ doesn't exist.

<Proof> On the contrary, assume that $\lim_{x\to a} f(x) = L$.

$\because f(x)g(x) = 1$ for all x and $\lim_{x\to a} g(x) = 0$

$$\therefore \lim_{x\to a}\{f(x)g(x)\} = 1$$

$$\parallel$$

$$(\lim_{x\to a} f(x))\cdot(\lim_{x\to a} g(x))$$

$$\parallel$$

$$L\cdot 0$$

$$\parallel$$

$$0$$

$\times$

Hence, $\lim_{x\to a} f(x)$ doesn't exist.

Section 2.4

(20) $\frac{\infty}{\infty}$ 不定型

$$\lim_{x\to\infty} \frac{\sqrt{2x+1}/x}{(x+4)/x} = \lim_{x\to\infty} \frac{\sqrt{\frac{2}{x}+\frac{1}{x^2}}}{1+\frac{4}{x}} = \frac{0}{1} = 0$$

(31)

$$\lim_{x\to 5^-} \frac{\overset{\to 25}{x^2}}{\underbrace{(x-5)}_{\to 0}\underbrace{(3-x)}_{\to -2}} = \infty$$

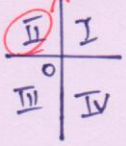
$(x<5)$

$\Rightarrow x-5<0$

(34)

$$\lim_{\theta\to(\frac{\pi}{2})^+} \frac{\overset{\to\frac{\pi}{2}}{\pi\theta}}{\underbrace{\cos\theta}_{\to 0}} = -\infty$$

$\theta\fallingdotseq\frac{\pi}{2}$ $(\theta>\frac{\pi}{2})$



$\Rightarrow \cos\theta<0$

(42)

$\because -1\le \sin x\le 1$ for all x

$\therefore \frac{-1}{x}\le \frac{\sin x}{x}\le \frac{1}{x}$ for all $x>0$

$\because \lim_{x\to\infty}\frac{1}{x}=0$ and $\lim_{x\to\infty}\frac{-1}{x}=0$

By Squeeze Theorem $\therefore \lim_{x\to\infty}\frac{\sin x}{x}=0$

(59) $\infty-\infty$ 不定型

$$\lim_{x\to-\infty}\left[\left(\sqrt{2x^2+3x}-\sqrt{2x^2-5}\right)\frac{\sqrt{2x^2+3x}+\sqrt{2x^2-5}}{\sqrt{2x^2+3x}+\sqrt{2x^2-5}}\right]$$

$$=\lim_{x\to-\infty}\frac{(2x^2+3x)-(2x^2-5)}{\sqrt{2x^2+3x}+\sqrt{2x^2-5}}$$

$$=\lim_{x\to-\infty}\frac{(3x+5)/x}{\left(\sqrt{2x^2+3x}+\sqrt{2x^2-5}\right)/x}$$

$\left(\because \text{For } x<0 \quad \sqrt{x^2}=-x \Rightarrow x=-\sqrt{x^2}\right)$

$$=\lim_{x\to-\infty}\frac{3+\frac{5}{x}}{-\sqrt{2+\frac{3}{x}}-\sqrt{2-\frac{5}{x}}} = -\frac{3}{2\sqrt{2}}$$